1.3.5 Compound Interest & Depreciation

Compound Interest

What is compound interest?

- Interest is a small percentage paid by a bank or company that is added onto an initial investment.
- Interest can also refer to an amount paid on a loan or debt; however, IB compound interest questions will always refer to interest on **investments**.
- **Compound interest** is where interest is paid on **both the initial investment** and any interest that has **already been paid**.
- The interest paid each time will increase as it is a percentage of a higher number.
- Compound interest will be paid in instalments in a given timeframe.
  - The interest rate, r, will be per annum.
  - This could be written as % p.a.
- Look out for phrases such as **compounding annually** or **compounding monthly**.

If $a\%$ p.a. is compounded monthly, then

$$\frac{a}{12}\%$$

will be paid each month.

How is compound interest calculated?

The formula for calculating compound interest is

$$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$$

where

- FV is the future value,
- PV is the present value,
- n is the number of years,
- k is the number of compounding periods per year,
- $r\%$ is the nominal annual rate of interest.

This formula is given in the formula booklet, so you do not have to remember it.

Common values of k

- Compounding annually means $k = 1$.
- Compounding half-yearly means $k = 2$.
- Compounding quarterly means $k = 4$.
- Compounding monthly means $k = 12$.

Examiner Tip

Your GDC will be able to solve some compound interest problems, so it is a good idea to make sure you are confident using it. However, you must also familiarise yourself with the formula and know where to find it in the formula booklet.

Worked Example

Kim invests MYR 2000 in an account that pays a nominal annual interest rate of 2.5%, compounded monthly.

Calculate the amount that Kim will have in her account after 5 years.

$$PV = 2000$$

$$r = 2.5$$

$$k = 12$$

$$n = 5$$

$$FV = 2000\left(1 + \frac{2.5}{100(12)}\right)^{12(5)}$$

$$FV = 2266.002\ldots$$

$$\boxed{FV \approx \text{MYR } 2270 \text{ (3 s.f.)}}$$

Depreciation

What is depreciation?

- Depreciation is when the **value** of something falls over time.
- The most common examples of depreciation are the value of cars and technology.
- If depreciation occurs at a constant rate, then it is **compound depreciation**.

How is compound depreciation calculated?

The formula for compound depreciation is

$$FV = PV\left(1 - \frac{r}{100}\right)^n$$

where

- FV is the future value,
- PV is the present value,
- n is the number of years,
- $r\%$ is the rate of depreciation.

This formula is **not** given in the formula booklet, but it is almost the same as the compound interest formula:

- with subtraction instead of addition,
- the value of k will always be 1.

Examiner Tip

You can use your GDC's finance solver or compound interest feature to solve most depreciation questions by entering the interest rate as a negative value.

Worked Example

Kyle buys a new car for AUD 14 999. The value of the car depreciates by 15% each year.

(a) Find the value of the car after 5 years.

$$FV = PV\left(1 - \frac{r}{100}\right)^n$$

$$PV = 14999$$

$$r = 15$$

$$n = 5$$

$$FV = 14999\left(1 - \frac{15}{100}\right)^5$$

$$FV = 6655.13\ldots$$

$$\boxed{FV \approx \text{AUD } 6660 \text{ (3 s.f.)}}$$

(b) Find the number of years and months it will take for the value of the car to be approximately AUD 9999.

$$FV = PV\left(1 - \frac{r}{100}\right)^n$$

$$9999 = 14999\left(1 - \frac{15}{100}\right)^n$$

$$9999 = 14999(0.85)^n$$

$$\frac{9999}{14999} = 0.85^n$$

Taking logarithms,

$$n = \frac{\ln\left(\frac{9999}{14999}\right)}{\ln(0.85)}$$

$$n = 2.49\ldots$$

Convert the decimal part to months:

$$0.49 \times 12 = 5.88$$

$$\boxed{\text{Approximately 2 years and 6 months}}$$